

1086-65-612

Fred J. Hickernell* (hickernell@iit.edu), Room E1-208, Dept. of Applied Mathematics, Illinois Institute of Technology, 10 W. 32nd St., Chicago, IL 60616. *Guaranteed Automatic Algorithms for Integration and Function Recovery.*

Numerical libraries contain automatic algorithms, where the work expended is *adaptively* adjusted to the difficulty of the problem, and the answer is provided to within the user-specified error tolerance. Examples include MATLAB's `quad` routine for integration and the `chebfun` toolbox. Often these automatic algorithms perform as advertised, but they can be fooled. In fact, James Lyness argued persuasively in a 1983 SIAM Review article that one could always fool numerical integration algorithms, even with integrands that do not look very strange. This talk presents a framework for automatic numerical algorithms with *rigorous guarantees*, something lacking for nearly all existing automatic algorithms. This framework is illustrated with examples for numerical integration and function recovery. The key is identifying a *cone* of input functions. This departs from typical error analyses that focus on balls of input functions. The algorithms presented here overcome the objections of Lyness because they violate his inherent assumptions. Information-based complexity theory provides sufficient conditions under which adaption does not help. In the setting studied here, these sufficient conditions are violated, and so adaption can, and does, help. (Received September 08, 2012)