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Jun Tao* (jtao68@yahoo.com), 14207 Eagle Mine Drive, Poway, CA 92064. *The Development of the Fundamental Theorem of Calculus by Using the Limit Approach Only.*

This paper describes how to develop the Fundamental Theorem of Calculus by only using the limit approach. Let $F(x)$ be differentiable and $f(x)$ be continuous on an interval $[a, b]$, where $F'(x) = f(x)$. Let's divide the interval $[a, b]$ into n pieces of small intervals whose width is h ($h = \frac{b-a}{n}$); then the following formula can be directly proved by using the limit approach:

$$\lim_{n \rightarrow \infty} [F'(a)h + F'(a+h)h + \cdots + F'(a+(n-1)h)h] = F(b) - F(a)$$

Since $f(x) = F'(x)$, we have $f(a)h = F'(a)h, \dots, f(a+(n-1)h)h = F'(a+(n-1)h)h$. Let's substitute them into the proved formula, we have:

$$\lim_{n \rightarrow \infty} [f(a)h + f(a+h)h + \cdots + f(a+(n-1)h)h] = F(b) - F(a)$$

Rewrite the formula as:

$$\int_a^b f(x) dx = F(b) - F(a)$$

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