

1096-34-1392      **Bruce Reznick\*** ([reznick@math.uiuc.edu](mailto:reznick@math.uiuc.edu)), Department of Mathematics, University of Illinois,  
1409 W. Green St., Urbana, IL 61801. *Every math major should know this crazy theorem.*

THM: The solutions of the equation

$$((y'')^{-2/3})''' = 0$$

are precisely the non-degenerate conic sections.

We shall give two old, elementary and comprehensible proofs.

This theorem appears in Sylvester (1886), although Cartan (1937) attributes it to Halphen (1870's). The essentially equivalent version

$$y''(40(y''')^3 - 45y''y''''y'''' + 9(y'')^2y''''') = 0,$$

which is valid for non-vertical lines as well, goes back to Monge (1809). More sophisticated interpretations of this result have been given by Lascoux (2006). (Received September 15, 2013)