Past research in the area of Gray codes has focused extensively on using this code to label vertices of graphs and to discover characteristics of these graphs. We continue in this direction but deviate slightly by creating edges between vertices which are not of Gray code distance 1 from each other, but instead of some other set integer distance. We then find characteristics of these new graphs. The majority of our results focus on distance 2 graphs, with some results for general distances. Our biggest result is on the structure of the graphs for any distance. We find that two isomorphic components result when the distance is even, as long as the distance is strictly larger than the string length, and that the graph will have only one component if the distance is odd and strictly greater than the string length. Other areas explored include the chromatic number, cliques, Hamiltonicity, and perfect matchings of the graphs. (Received September 09, 2014)