Replacing square grids with triangular grids in Robertson and Seymour.

The $n \times n$-grid is the canonical example of a planar graph of treewidth $n$, and it plays a pivotal role in setting up the base case in Robertson and Seymour’s proof of the Graph Minors Theorem. Their choice of the square grid relies on both its symmetry and simplicity. However, the $n \times n$-grid is far from being a minimal planar graph of treewidth $n$ with respect to the minor relation.

We show that the triangular grids provide another family of graphs which realize every integer treewidth, and we characterize minimal structure common to both the triangular grids and the square grids. Moreover, we seek to use this structure to improve both algorithmic and theoretical lower bounds on treewidth. (Received September 12, 2014)