Are All 2-connected Maximal Non-Hamiltonian Graphs Spanned by $\theta$ Graphs? Preliminary report.

One way to study Hamiltonicity of graphs is to consider (edge-)maximal non-Hamiltonian graphs. For instance, Ore’s sufficient condition for a graph to be Hamiltonian can be rephrased as saying that in any maximal non-Hamiltonian graph of order $n$, there are two nonadjacent vertices whose degrees sum to less than $n$. In fact, the Bondy–Chvatal Theorem says that this property holds for every pair of nonadjacent vertices.

It is easy to show that every maximal non-Hamiltonian graph of order at least 3 is spanned by a figure-8 graph (the union of two cycles sharing a point, where we allow each cycle to degenerate to an edge). Now maximal non-Hamiltonian graphs with cut-vertices are easily classified, so we can restrict our attention to 2-connected maximal non-Hamiltonian graphs. I conjecture that every such graph is spanned by a $\theta$ graph (a subdivision of $K_{1,1,2}$). I have shown that this holds for all such graphs of order at most 20 and have proved a number of properties that a potential counterexample would have to possess. (Received September 16, 2014)