Accessibility numbers in the sandpile monoid of a directed graph. Preliminary report.

Let $\Gamma = (V, E)$ be a directed graph with a global sink $s$. A sandpile $c$ is a vector of non-negative integers indexed by $V \setminus s$. Given a sandpile $c$, if $c(v) < \text{outdeg}(v)$, for all non-sink vertices $v$ then $c$ is stable; otherwise, $c$ is unstable. In the latter case, $c$ may be stabilized by a sequence of vertex topplings where an unstable vertex $v$ topples sending a grain of sand through each of its out-edges. A stable sandpile $c$ is accessible from a sandpile $b$ if one can reach $c$ from $b$ by a series of sand additions and topplings. The accessibility number of a stable sandpile $c$ is the number of stable sandpiles that access $c$. This leads to the accessibility polynomial of $\Gamma$

$$A(x) = \sum_{i=1}^{m} a_i x^i,$$

where $m$ is the number of stable sandpiles in $\Gamma$ and $a_i$ is the number of stable sandpiles with accessibility number $i$. The coefficient $a_m$ is thus the number of sandpiles that are accessible by all other stable sandpiles. In this talk we introduce the accessibility polynomial and discuss some of its properties. For example, $a_m$ equals the number of spanning trees of $\Gamma$ directed to $s$. (Received September 02, 2014)