Ira M. Gessel* (gessel@brandeis.edu) and Yan Zhuang. Counting permutations with even valleys and odd peaks.

We find the exponential generating function for permutations with all valleys even and all peaks odd, answering a question posed by Liviu Nicolaescu. The generating function is

\[
\left(1 - E_1 x + E_3 \frac{x^3}{3!} - E_4 \frac{x^4}{4!} + E_6 \frac{x^6}{6!} - E_7 \frac{x^7}{7!} + \cdots\right)^{-1},
\]

where \( \sum_{n=0}^{\infty} E_n x^n/n! = \sec x + \tan x \), which resembles David and Barton’s generating function

\[
\left(1 - x + \frac{x^3}{3!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \frac{x^7}{7!} + \cdots\right)^{-1},
\]

for permutations with no increasing runs of length 3 or more.

Following Dennis Chebikin, we define an alternating descent of a permutation to be an odd descent or an even ascent, and we define an alternating run to be a maximal consecutive subsequence with no alternating descents. Then the permutations we want to count are those with no alternating runs of length 3 or more.

Using noncommutative symmetric functions, we explain the similarity of (1) and (2) as a special case of a very general connection between generating functions for permutations by increasing runs and by alternating runs. (Received September 03, 2014)