The classical theorem of Vizing states that every graph of maximum degree $d$ admits an edge-coloring with at most $d + 1$ colors. Furthermore, as it was earlier shown by König, $d$ colors suffice if the graph is bipartite.

We investigate the existence of measurable edge-colorings for graphings. A graphing is an analytic generalization of a bounded-degree graph that appears in various areas, such as sparse graph limits and orbit equivalence theory. We show that every graphing of maximum degree $d$ admits a measurable edge-coloring with $d + O(\sqrt{d})$ colors; furthermore, if the graphing has no odd cycles, then $d + 1$ colors suffice. In fact, if a certain conjecture about finite graphs that strengthens Vizing’s theorem is true, then our method will show that $d + 1$ colors are always enough. (Received September 06, 2014)