Sets Characterized by Sums and Differences in Dilating Polytopes.

In 2007, Hegarty showed that for any prescribed $s, d \in \mathbb{N}_0$, the proportion $\rho_{s,d}^n$ of subsets of $\{0, \ldots, n\}$ that are missing exactly $s$ sums in $\{0, \ldots, 2n\}$ and exactly $2d$ differences in $\{-n, \ldots, n\}$ also remains positive in the limit. We consider the following question: are such sets, characterized by their sums and differences, similarly ubiquitous in higher dimensional spaces? Let $P$ be a polytope in $\mathbb{R}^D$ with vertices in $\mathbb{Z}^D$, and let $\rho_{s,d}^n$ now denote the proportion of subsets of $L(nP)$ that are missing exactly $s$ sums in $L(nP) + L(nP)$ and exactly $2d$ differences in $L(nP) - L(nP)$. It turns out the geometry of $P$ has a significant effect on the limiting behavior of $\rho_{s,d}^n$. We define a geometric feature of polytopes called local point symmetry, and show that $\rho_{s,d}^n$ is bounded below by a positive constant as $n \to \infty$ if and only if $P$ is locally point symmetric. We also show that the proportion of subsets in $L(nP)$ missing exactly $s$ sums and at least $2d$ differences remains positive in the limit, independent of the geometry of $P$. A corollary of these results is that if $P$ is point symmetric, the proportion of sum-dominant subsets of $L(nP)$ also remains positive in the limit. (Received September 15, 2014)