Arithmetic Progressions on Curves.

The set \( \{1, 25, 49\} \) is a 3-term collection of integers which forms an arithmetic progression; the common difference is 24. Hence the set \( \{(1, 1), (5, 25), (7, 49)\} \) is a 3-term collection of rational points on the parabola \( y = x^2 \) whose \( y \)-coordinates form an arithmetic progression. Similarly, the set \( \{6, 12, 18\} \) is a 3-term collection of integers which also forms an arithmetic progression; the common difference is 6. Hence the set \( \{(6, 3), (12, 39), (18, 75)\} \) is a 3-term collection of rational points on the elliptic curve \( y^2 = x^3 - 207 \) whose \( x \)-coordinates form an arithmetic progression. Are there other examples such as these? What is the longest progression of rational points on either a quadratic or cubic curve such that either the \( x \)- or \( y \)-coordinates form an arithmetic progression? In this talk, we give a survey on what’s known about arithmetic progressions on algebraic curves. We introduce elliptic curves as a means to show the non-existence of certain arithmetic progressions. We also introduce bielliptic curves in order to settle conjectures of Saraju P. Mohanty. (Received August 28, 2014)