Michael Kerckhove* (mkerckho@richmond.edu). Unitary Equivalence of Rank $k$ Partial Isometries on $\mathbb{C}^n$ via the Stiefel Manifold $S(k,n)$.

A rank $k$ partial isometry on $\mathbb{C}^n$ factors as $A = VW^*$ for two $n$-by-$k$ matrices $V$ and $W$ having orthonormal columns. For such matrices denote $\text{col}(V)$ by $p(V)$. The group $U(n) \times U(k)$ acts on matrices $V$, elements of $S(k,n)$, by $(U_n, U_k) \cdot V = U_n V U_k$, inducing an action of $U(n)$ on $p(S(k,n))$. We show that

1) With $E = \begin{bmatrix} I_k & 0 \\ 0 & 0 \end{bmatrix}$, $VW^*$ is unitarily equivalent to a partial isometry $XE^*$, and if $XE^* \sim YE^*$ then $p(Y) = h.p(X)$ for some $h \in H = \left\{ \begin{bmatrix} U_k & 0 \\ 0 & U_{n-k} \end{bmatrix} \right\}$. For $h \in H$ let $\pi(h) = U_k$.

2) With $H_X$ the isotropy subgroup at $p(X)$, the map $\phi : H_X \to U(k)$ given by $hX = X\phi(h)$ is a homomorphism and $XE^* \sim XU_k' E^*$ if and only if $U_k' = \phi(h)\pi(h)$ for some $h \in H_X$.

Together these results provide a mapping from the set of partial isometries unitarily equivalent to $A = VW^*$ into the Stiefel manifold $S(k,n)$ whose image coincides with an action of $H$. The orbit space for this group action then characterizes equivalence classes of rank $k$ partial isometries on $\mathbb{C}^n$. (Received September 16, 2014)