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Michael Kerckhove* (mkerckho@richmond.edu). *Unitary Equivalence of Rank k Partial Isometries on \mathbb{C}^n via the Stiefel Manifold $S(k, n)$.*

A rank k partial isometry on \mathbb{C}^n factors as $A = VW^*$ for two n -by- k matrices V and W having orthonormal columns. For such matrices denote $\text{col}(V)$ by $p(V)$. The group $\mathcal{U}(n) \times \mathcal{U}(k)$ acts on matrices V , elements of $S(k, n)$, by $(U_n, U_k).V = U_n V U_k$, inducing an action of $\mathcal{U}(n)$ on $p(S(k, n))$. We show that

1) With $E = \begin{bmatrix} I_k \\ 0 \end{bmatrix}$, VW^* is unitarily equivalent to a partial isometry XE^* , and if $XE^* \sim YE^*$ then $p(Y) = h.p(X)$

for some $h \in H = \left\{ \begin{bmatrix} U_k & 0 \\ 0 & U_{n-k} \end{bmatrix} \right\}$. For $h \in H$ let $\pi(h) = U_k$.

2) With H_X the isotropy subgroup at $p(X)$, the map $\phi : H_X \rightarrow \mathcal{U}(k)$ given by $hX = X\phi(h)$ is a homomorphism and $XE^* \sim XU'_k E^*$ if and only if $U'_k = \phi(h)\pi(h)$ for some $h \in H_X$.

Together these results provide a mapping from the set of partial isometries unitarily equivalent to $A = VW^*$ into the Stiefel manifold $S(k, n)$ whose image coincides with an action of H . The orbit space for this group action then characterizes equivalence classes of rank k partial isometries on \mathbb{C}^n . (Received September 16, 2014)