

1106-16-1696

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Karl-Franzens-Universität Graz, Heinrichstraße 36, 8010 Graz, Austria. *A semigroup-theoretical  
view of direct-sum decompositions over HNP rings.*

We study the arithmetic of direct sum decompositions of finitely generated modules over a hereditary noetherian prime ring. Let  $R$  be a (non-artinian) HNP ring, and let  $\mathcal{C}$  be the class of f.g. projective right  $R$ -modules. The recent monograph by Levy and Robson describes the structure of  $\mathcal{C}$  by means of simple  $R$ -modules (up to stable isomorphism of modules, and up to isomorphism if the uniform dimension of the modules is at least two). Denote by  $H$  a set of representatives of (stable) isomorphism classes of  $\mathcal{C}$ . Then the direct sum operation induces the structure of a commutative semigroup on  $H$ . This semigroup carries essentially all information about direct sum decompositions in  $\mathcal{C}$ , and hence the study of the arithmetic of direct sum decompositions in  $\mathcal{C}$  can be reduced to the study of the factorization theory of the semigroup  $H$ . Using the results of Levy and Robson, we obtain a purely algebraic description of  $H$ . We investigate the half-factorial monoids obtained in this way, and are able to determine various arithmetic invariants such as the catenary and tame degree. This yields a description of the arithmetic of direct-sum decompositions in  $\mathcal{C}$  in terms of the tower structure of  $R$ . (Received September 15, 2014)