Model categories have been widely used since their introduction by Quillen in 1967, and although many techniques exist for constructing model categories the most fundamental question remains open: given a bicomplete category $C$ together with a subcategory $\mathcal{W}$, when does there exist a model structure on $C$ with $\mathcal{W}$ as the subcategory of weak equivalences? This question is fundamentally important, as model categories do not generally arise naturally “in the wild”; instead, one generally has a category with a subcategory of weak equivalences, and must construct the model structure by hand. Although this question is very difficult in general, it turns out that when $C[\mathcal{W}^{-1}]$ is a preorder the question can often be answered. We present some techniques for dealing with this question in general and, in the case when $C$ is small, give necessary and sufficient conditions for the existence of a model structure. (Received September 11, 2014)