An old dream of mine has been to prove that almost all continued fractions $K(a_n/b_n)$ with complex elements $a_n$, $b_n$ converge. It is easy to come up with divergent continued fractions, but they are rather special, such as limit periodic continued fractions $K(a_n/b_n)$ of elliptic type, or $(a_n/b_n b_{n-1}) \to \infty$ too fast. But a statement like that would naturally have to depend on the measure on the space of continued fractions. What I had in mind was some kind of sensible, natural measure.

A random continued fraction is a continued fraction where the elements $(a_n, b_n)$ are picked randomly and independently from a probability distribution on $\mathbb{C} \times \mathbb{C}$. What are the chances that $K(a_n/b_n)$ converges? This is not quite the same question, but a connection was provided by Furstenberg already in 1963 in a different setting. Still, once on the track, it is easy to find other results which are of help, such as the Borel-Cantelli Lemma, Kolmogorov’s 0-1 theorem and a lemma by L. Arnold.

With these tools it is possible to prove that under mild conditions on the measure $\mu$ on $\mathbb{C} \times \mathbb{C}$, the $\mu$-random continued fraction converges with probability 1. It is a miracle that continued fractions are so cooperative... (Received September 15, 2014)