Mean dimension and metric mean dimension are dynamical invariants of an action of a group on a compact metrizable space. They were defined for the case when the group is amenable by Lindenstrauss and Weiss, and has been extended to the sofic case by Li. Metric mean dimension can be thought of as a dynamically version of dimension, and is an analogue of entropy for “large” spaces. For example, the metric mean dimension of a Bernoulli shift is the dimension of the base. In this work, we are concerned with metric mean dimension in the case of algebraic actions which are actions of a group $\Gamma$ by automorphisms on a compact, metrizable, abelian group $X$. When $\Gamma$ is sofic, we relate the metric mean dimension of $X$ to a quantity called the von Neumann-Lück rank of the dual of $X$. The von Neumann-Lück rank of a $\mathbb{Z}(\Gamma)$-module $A$ may be regarded as the von Neumann dimension of a certain Hilbert space representation associated to $A$. This work is a partial generalization of results due to Li-Liang in the amenable case, and is part of several instances in which invariants of algebraic actions are related to $L^2$-invariants of $\mathbb{Z}(\Gamma)$ modules. (Received September 15, 2014)