Does Lipschitz imply fast convergence of a diffusion-smearred function to itself as time goes to 0? Preliminary report.

For a diffusion semigroup $A_t$ on some function space, it is natural to investigate the relationship between smoothness of a function $f$ and the speed of convergence of $A_t f$ to $f$ as $t \to 0^+$. Recently, R.R. Coifman and W.E. Leeb have proposed a family of multi-scale diffusion metrics on the underlying measure space. To show that Lipschitz implies fast convergence of $A_t f$ to $f$, they assume a certain rate of decay, as $t \to 0^+$, of the expected value of the distance from a generic point $x$ to another point in the underlying measure space, with respect to the probability density associated with the diffusion at time $t$. It would be useful to know more about when this assumption holds.

In our work, we show that we can replace the $L_1$ metric of probability densities at time $t$ used by Coifman and Leeb to construct their diffusion distance family, by an appropriate, automatically normalized $L_2$ metric using square roots of the probability densities. This metric, or rather its square, leads to more explicit computation of an integral which is relevant to establishing the extra assumption above. Moreover, power decay of this integral with respect to our metric is equivalent to power decay using the original $L_1$ metric. (Received September 16, 2014)