Variational inequality and surjectivity of noncoercive operators and application to nonlinear parabolic problems. Preliminary report.

Let $X$ be a real reflexive locally uniformly convex Banach space with locally uniformly convex dual $X^*$. New variational inequality and surjectivity results are obtained for noncoercive operators of the type $T + A + S$ where $T : X \supseteq D(T) \rightarrow 2^{X^*}$ and $A : X \supseteq D(A) \rightarrow 2^{X^*}$ are maximal monotone and $S : X \supseteq D(S) \rightarrow 2^{X^*}$ is bounded pseudomonotone. A positive answer for Nirenberg’s problem is included for quasimonotone expansive mappings. The results are new and improve the corresponding theory concerning surjectivity of coercive operators of monotone type. The theory developed herein is applied to study existence of generalized solution(s) in $X = L^p(0, T; W^{1,p}_0(\Omega))$ (with suitable $p > 1$) of the parabolic problem

$$\begin{cases}
\frac{\partial u}{\partial t} - \triangle_p u + g(x, u, \nabla u) = f & \text{in } (0, T) \times \Omega \\
u(t, x) = 0 & \text{in } (0, T) \times \partial \Omega
\end{cases}$$

where $f$ is a given function, $\Omega$ is nonempty, bounded and open subset of $\mathbb{R}^N$ and $g : \Omega \times \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}$ satisfies certain conditions.

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