One of the most important open problems on Einstein homogeneous manifolds is the Alekseevskii conjecture, which states that any connected, non-compact, homogeneous Einstein manifold must be diffeomorphic to a Euclidean space. Up to now, it was known that the conjecture was true only up to dimension 5, and in dimension 6 when the transitive group is not semisimple. The purpose of this talk is to show that the conjecture holds up to dimension 8 (excluding the case of semisimple Lie groups), and up to dimension 10 when the transitive group is not semisimple. (Received August 29, 2014)