In percolation theory, a major problem is finding the percolation threshold at which an infinite open cluster begins to exist. Exact percolation threshold solutions are extremely rare. In 1990, Wierman introduced the substitution method to determine rigorous bounds for percolation thresholds. The substitution method compares two subgraphs (substitution regions) from two lattices so that the probability measure of one subgraph is stochastically larger than the probability measure of the other subgraph.

To prove the stochastic ordering, one straightforward method is by listing all the upsets and comparing their corresponding probabilities with respect to the two probability measures. An alternative choice is by the connection between stochastic ordering and network-flow, introduced by Preston. With a carefully designed network flow structure, one is able to prove stochastic ordering by solving for the max-flow value. This approach has been proved to be more efficient, but has the disadvantage of a relatively large numerical error. We will discuss some feasible improvements to reduce the error and provide latest results of the bounds for the percolation threshold of some common Archimedean lattices.

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