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Hannah Alpert, Jesús A. De Loera, Susan Margulies, Michael Pernpeintner, Eric Riedl, David Rolnick* (drolnick@math.mit.edu), **Gwen Spencer, Despina Stasi** and **Jonathan Swenson**. *Algebraic algorithms for hard combinatorial problems: Graph coloring*.

A remarkable number of difficult combinatorial problems can be rephrased in terms of polynomial systems, an approach that Alon has termed the "polynomial method". Accordingly, there is great interest in algorithms that find explicit or approximate solutions to combinatorial polynomial systems in a reasonable amount of time. We consider the complexity of finding the chromatic number of a graph by solving an associated system of polynomial equations. We identify graphs for which the method of Gröbner bases yields a fast solution. We also consider the difficulty of proving that the system is infeasible using the Hilbert Nullstellensatz, by providing lower bounds on the minimum degree of a Nullstellensatz certificate. Finally, we show how graph coloring implies that certain problems for Gröbner bases are NP-hard. (Received September 17, 2014)