

1106-81-756

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90095-1555. *Quantum Variables*.

Although the mathematical notions of quantized variables were first considered in the late twenties by von Neumann, it wasn't until the forties that such researchers as Segal, Kadison, Singer, and Dixmier realized their importance in mathematics. Finite density matrices provide the simplest examples. In this model, one replaces probability measures $p = (p_1, \dots, p_n)$, where $(p_j \geq 0, \sum p_j = 1)$ by density matrices $\rho = [\rho_{ij}]$ which are positive semidefinite, and for which $\text{tr}(\rho) = \sum \rho_{ii} = 1$. The parallel between these quantities is best understood if one regards them as states on the C^* -algebras \mathbb{C}^n and \mathbb{M}_n , respectively. From the point of view of thermodynamics, the entropies $H(p) = \sum p_j \log p_j$ and $S(\rho) = \text{tr}(\rho \log \rho)$ provide important invariants. Subsequently, it was discovered that the seemingly unnatural relative entropies $H(p||q) = \sum p_j \log p_j - p_j \log q_j$ and $S(\rho||\sigma) = \text{tr}(\rho \log \rho - \rho \log \sigma)$ are even more fundamental. The remarkable parallels between these quantities and their applications to such areas as classical and quantum information theory will be reviewed. (Received September 05, 2014)