We study the following question proposed by Mattila in 1998: what are the self-similar subsets of the middle-third Cantor set $C$? For any non-trivial self-similar subset $F$ of $C$, we show that any linear generating IFS of $F$ should consist of similitudes with contraction ratios $1/3^n$. Furthermore, we provide a necessary and sufficient condition to characterize all self-similar subsets of $C$. A very simple criterion is formulated to characterize self-similar subsets of $C$ with equal contraction ratio. A finite algorithm is provided to generate all self-similar subsets of $C$ with pre-given contraction ratios. (Received September 14, 2014)