The zeta function is an \( L \)-function whose zeros encode information about the primes. Similarly, the zeros of \( L \)-functions associated to elliptic curves, modular forms, and Dirichlet characters (among others) contain information about those objects. In this vein, the order of vanishing at the point \( s = 1/2 \) is particularly important.

We can study that order of vanishing using a statistic called 1-level density. It is an appealing statistic because of the Katz-Sarnak conjecture that the average one level density of a family of \( L \)-functions can be calculated without knowing any zeros in the family. Confirmed for certain families of holomorphic cusp forms, the conjecture states that the average one level density of a family depends on the family’s symmetry group and a chosen test function. It is then natural to ask which test function gives the best results on vanishing.

Iwaniec, Luo, and Sarnak found the optimal test functions whose Fourier transforms are supported in \((-2,2)\). We generalize their analysis of Fredholm operators to find the optimal test function for any finite interval of support. Additionally, we determine the optimal test functions for the 2 and higher level densities, which yield better results on order vanishing at the central point. (Received September 16, 2014)