The diameter of polytopes is related to the efficiency of the simplex algorithm used in linear optimization. We will briefly describe the connection. Our approach is to study the diameters of polytopes by studying subset partition graphs, a certain type of abstract polytope. The vertex sets of these graphs satisfy a simple additional combinatorial property related to partitioning a given fixed set. After presenting a complete introduction to polytope and abstract polytope topics relevant for this talk, we present recent lower bounds on diameters for graphs of abstract polytopes and several new results relating the various previous notions of abstract polytopes and their graphs. The talk is completely self-contained and includes collaborations with Tristram C. Bogart and J. Mackenzie Gallagher. (Received September 03, 2014)