The most common category considered in (undirected) graph theory is a category where graphs are defined as having at most one edge incident to any two vertices and at most one loop incident to any vertex. The morphisms are usually described as a pair of functions between the vertex sets and edge sets that respect edge incidence. We will relax these conditions to allow multiple edges to be incident to any two vertices, multiple loops to be incident to any vertex, and morphisms will be allowed to map edges to vertices, but they must still preserve edge incidence. With combinations of these three relaxations we define five categories of graphs.

We follow the lead and spirit of F. W. Lawvere’s groundbreaking characterization of the Category of Sets and Functions and D. Schlomiuk’s characterization of the Category of Topological Spaces and Continuous Functions. In both characterizations, a list of elementary axioms are provided so that when combined with a higher order axiom a functor equivalence between the axiomatically defined category and the concrete category is formed. We provide such an elementary theory for the five categories of graphs. (Received September 15, 2014)