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Ramsey Theory Over Imaginary Quadratic Number Fields.

Optimal bounds in Ramsey theory usually require the construction of the largest sets not possessing a given property. One problem which has been investigated by many concerns the construction of the largest possible subset of $\{1, 2, \dots, n\}$ (as $n \rightarrow \infty$) that are free of 3-term geometric progressions. Building on the significant recent progress in constructing such large sets, we consider higher dimensional analogues.

Specifically, let $\mathbb{Z}[i] = \{a + ib, a, b \in \mathbb{Z}\}$ (with $i = \sqrt{-1}$) be the Gaussian integers. We derive sets of Gaussian integers that avoid 3-term geometric progressions by utilizing the relationship between norms and powers of primes in the norms' factorization. Motivated by Rankin's canonical greedy set over the integers, we construct a set of Gaussian integers avoiding Gaussian integer ratios in a similar fashion, and compute its density using Euler products and the Riemann zeta function. We also establish upper and lower bounds on the maximum upper density of such sets. Finally, we discuss our extensions to other quadratic number fields, and the dependence of the density on the structure of the number field (in particular, on the norm and class number). (Received September 16, 2014)