David Milovich* (ultrafilter@gmail.com), Dept. of Mathematics and Physics, 5201 University Blvd., Laredo, TX 78041. Amalgamating many Boolean algebras.

Δ-systems of overlapping Boolean algebras always extend to a common Boolean algebra, but non-Δ-systems may not: we might have $x <_A y <_B z <_C x$. Non-Δ-systems are unavoidable when constructing a Boolean algebra (or any structure) of size $\geq \aleph_3$ as a directed union of countable structures. We prove a nontrivial sufficient condition for $n$ overlapping Boolean algebras to have a common extension. Along the way, we prove an $n$-ary version of the Interpolation Theorem of propositional logic.

Using also the set-theoretic technique of long $\omega_1$-approximation sequences (also known as Davies sequences), we obtain a flexible method of constructing (in ZFC) arbitrarily large Boolean algebras as direct limits of countable Boolean algebras. Our main application is a Boolean algebra of size $\aleph_n$ with the $n$-ary FN but not the $(n+1)$-ary FN where the $n$-ary FN is a higher-arity variant of the Freese-Nation property.

Our techniques also yield a new characterization of projective Boolean algebras that implies purely finitary facts about Vietoris hyperspace and symmetric power functors in the category of finite discrete spaces. (Received September 16, 2016)