Soon after its discovery by Frege and others, Naïve Set Theory (NST) was known to be paradoxical, giving rise to a wide variety of paradoxes, such as that of Russell. Several attempts to found NST using non-classical logics have so far obtained partial results. In this paper we provide a mathematically principled, motivated approach that makes sense of the set-theoretic paradoxes within the theory itself. By addressing the paradoxes substructurally—that is, within the very notion of proof itself—we retain proving ability while increasing expressiveness permitted in the theory beyond what is possible in the classical conception. We use properties of proof normalization in ways suggested by Prawitz, Hallnäs, and Ekman to ensure our system is coherent, and discuss notions of implication within the formal system.

This research was in part supported by the New Zealand Marsden Fund, and the Marie Curie IRSES (FP7) project Correctness by Construction. (Received September 18, 2016)