A theta graph is a graph consisting of three internally disjoint paths with common end vertices. By considering a BFS tree in a graph, it is not difficult to prove that if $G$ is a graph of order $n$ with minimum degree 3, then $G$ contains a theta subgraph of order at most $6 \log_2 n$. Note that the minimum degree condition is sharp, and there exists a graph of order $n$ with average degree 3 which does not contain a theta subgraph of order $o(n)$.

In this talk, we consider slightly weaker conditions, which ensure the existence of small theta subgraphs.

(1) Let $\alpha > 0$ and let $G$ be a graph of order $n$ with average degree at least $3 + \alpha$. Then, $G$ contains a theta subgraph of order at most $(\frac{9}{\alpha} + 3) \log_2 n$.

(2) Let $\beta > 0$ and let $G$ be a graph of order $n$ without isolated vertices. For $d \in \{1, 2\}$, let $n_d$ denote the number of vertices of degree $d$ in $G$. If $4n_1 + 3n_2 \leq (1 - \beta)n$, then $G$ contains a theta subgraph of order at most $(\frac{6}{\beta} + 1)(6 \log_2 n + 1)$.

These results enable us to prove that every large enough graph with minimum degree at least $2k + 1$ contains $k$ vertex-disjoint isomorphic theta subgraphs.

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