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Kylie Hess (hessko@rose-hulman.edu), **Jeremy Rouse** (rouseja@wfu.edu), **Emily Stamm*** (emstamm@vassar.edu) and **Terrin Warren** (terrinarwarren@gmail.com). *When is $a^n + 1$ the sum of two squares?* Preliminary report.

Using Fermat's two squares theorem and properties of cyclotomic polynomials, we prove assertions about when numbers of the form $a^n + 1$ can be expressed as the sum of two integer squares. We prove that $a^n + 1$ is the sum of two squares for all $n \in \mathbb{N}$ if and only if a is a perfect square. We also prove that for $a \equiv 0, 1, 2 \pmod{4}$, if $a^n + 1$ is the sum of two squares, then $a^\delta + 1$ is the sum of two squares for all $\delta|n$, $\delta > 1$. Using Aurifeuillian factorization, we show that if a is a prime and $a \equiv 1 \pmod{4}$, then there are either zero or infinitely many odd n such that $a^n + 1$ is the sum of two squares. When $a \equiv 3 \pmod{4}$, we define m to be the least positive integer such that $\frac{a+1}{m}$ is the sum of two squares, and prove that if $a^n + 1$ is the sum of two squares for any odd integer n , then $a^m + 1$ and $\frac{n}{m}$ are both sums of two squares. (Received August 05, 2016)