We develop the asymptotic formulas for correlations

$$\sum_{n \leq x} f_1(P_1(n))f_2(P_2(n)) \cdots f_m(P_m(n))$$

where $f_1, \ldots, f_m$ are bounded “pretentious” multiplicative functions, under certain natural hypotheses. We then deduce several desirable consequences: first, we characterize all multiplicative functions $f : \mathbb{N} \rightarrow \{-1, +1\}$ with bounded partial sums. This answers a question of Erdős from 1957 in the form conjectured by Tao. Second, we show that if the average of the first divided difference of multiplicative function is zero, then either $f(n) = n^s$ for $\text{Re}(s) < 1$ or $|f(n)|$ is small on average. This settles an old conjecture of Kátai. Finally, we discuss multidimensional analog of the results above and its applications to the study of Gowers norms of multiplicative functions. (Received September 18, 2016)