Replacing each nonzero entry of a matrix with the symbol * gives its zero-nonzero pattern. We may ask whether this combinatorial object captures any information about the eigenvalues of the matrix. Sometimes there exists a multiset of eigenvalues that can be ruled out for every matrix with that pattern; we say that the pattern fails to be spectrally arbitrary. It has been conjectured that an \( n \times n \) spectrally arbitrary pattern must have at least \( 2n \) nonzero entries.

The combinatorics of a particular zero-nonzero pattern (perhaps described by a digraph) may imply algebraic conditions on the coefficients of each characteristic polynomial belonging to a matrix with that pattern. Certain such algebraic conditions imply that the pattern cannot be spectrally arbitrary.

Is there a succinct set of algebraic conditions of this sort that apply to every pattern with fewer than \( 2n \) nonzero entries? We show that the answer is ‘yes’ for \( n \leq 6 \), and show that the question for \( n = 7 \) can be narrowed down to a very small number of patterns. (Received September 18, 2016)