The eigenvalues of complex Hermitian matrices are real, but what if the matrix is quaternionic? We consider several eigenvalue problems for Hermitian matrices over both the quaternions and octonions, showing to what extent it is possible to recover the properties expected by analogy with the complex case. There are nevertheless some interesting surprises along the way. Foremost among them is the fact that the octonionic projective space $\mathbb{O}P^2$ in fact consists of quaternionic elements. (Received August 12, 2016)