We look at the construction of $n$-dimensional hypercubes from $(n-1)$-dimensional hypercubes and look at various ways of visualizing the first five dimensions of hypercubes using 2D and 3D modeling. We explore the relationship between the external dimension of a hypercube and its internal elements when increasing either the external dimension or internal dimension by one. Let $X_{n,m}$ be the number of $m$-dimensional faces inside of an $n$-dimensional hypercube. We prove, by induction on $k=n+m$, the following recursive formula:

$$X_{n,m} = 2X_{n-1,m} + X_{n-1,m-1}$$

We explore the symmetric group of the $n$-dimensional hypercube, and discuss the relationship between the internal makeup of an $n$-dimensional hypercube and the size of the hypercube’s symmetric group. We then describe the alteration in the size of the multiplication table if different dimensions are allowed during the hypercube’s rotations. Last, we will examine the relationship between the multiplication table of a square and higher dimensional hypercubes. (Received September 19, 2016)