Let $G$ be a finite group. A character $\chi \in \text{Irr}(G)$ is called rational if $\chi(g) \in \mathbb{Q}$ for every $g \in G$, and an element $g \in G$ is called rational if $\chi(g) \in \mathbb{Q}$ for every $\chi \in \text{Irr}(G)$. If $g \in G$ is rational then we say the conjugacy class $cl_G(g)$ is rational.

Write $\text{Irr}_\mathbb{Q}(G)$ and $\text{Cl}_\mathbb{Q}(G)$, respectively, for the sets of rational irreducible characters and rational conjugacy classes of $G$. Extending work of Navarro-Tiep (2008) we show that when $G$ is non-solvable either $|\text{Irr}_\mathbb{Q}(G)| = 3$ if and only if $|\text{Cl}_\mathbb{Q}(G)| = 3$ or else the composition factors of $G$ are under very tight control. (Received September 19, 2016)