Second-order conformal quantum superintegrable systems in 2 dimensions are Laplace equations on a manifold with an added scalar potential and 3 independent 2nd order conformal symmetry operators. They encode all the information about 2D Helmholtz or time-independent Schrödinger superintegrable systems in an efficient manner: Each system admits a quadratic symmetry algebra and is multiseparable. The separation equations comprise all the types of hypergeometric and Heun equations in full generality, and they yield all of the 1D Schrödinger exactly solvable and QES systems related to the Heun operator. The different systems are related by Stäckel transforms, by the symmetry algebras and by Böcher contractions of the conformal algebra $so(4, C)$ to itself, which enables all systems to be derived from a single generic one. Distinct separable bases for a Laplace system are related by interbasis expansion coefficients, themselves special functions, such as the Wilson polynomials. Applying Böcher contractions to expansion coefficients for ES systems one can derive the Askey scheme for hypergeometric orthogonal polynomials. This approach facilitates a unified view of special function theory, incorporating hypergeometric and Heun functions in full generality. (Received September 06, 2016)