I’m going to talk about a transform that establishes a one-to-one correspondence between Verblunsky coefficients and two sequences \( \{ g_n \} \) and \( \{ r_n \} \) of real numbers such that

\[ 0 < g_n < 1, \quad -\infty < r_n < \infty. \]

As a matter of fact, the transform was introduced by Wall and it can be thought of as a transform between Schur functions, Carathéodory functions, and Nevanlinna functions. So, to be more precise the goal of the talk is to demonstrate that the sequences \( \{ g_n \} \) and \( \{ r_n \} \) also give rise to a very special class of linear pencils of Jacobi matrices. Consequently, the Wall transform shows that the theory of polynomials orthogonal on the unit circle is just a tip of the iceberg of linear pencils of Jacobi matrices. In particular, since orthogonal polynomials on the unit circle are very well studied, the transform in question helps to build up a certain intuition towards the general theory of linear pencils of Jacobi matrices (e.g. behavior of the entries from the properties of the corresponding measure and vice versa).

Finally, using the developed framework I will show how one can handle Cauchy-type distributions on the real line, which cannot be dealt with by means of orthogonal polynomials on the real line due to the non-existence of moments. (Received September 12, 2016)