Uniqueness results for classes of semipositone p-Laplacian problems.

We consider steady state reaction diffusion equations on the exterior of a ball, namely, boundary value problems of the form:

$$
\begin{cases}
-\Delta_p u = \lambda K(|x|)f(u) \quad \text{in } \Omega_E, \\
Bu = 0 \quad \text{on } |x| = r_0, \\
u \to 0 \quad \text{when } |x| \to \infty,
\end{cases}
$$

where $\Delta_p z := \text{div}(\|\nabla z\|^{p-2}\nabla z)$, $1 < p < n$, $\lambda > 0$, $\Omega_E := \{x \in \mathbb{R}^n \mid |x| > r_0 > 0\}$, and $Bu \equiv u$ or $Bu \equiv \frac{\partial u}{\partial n} + c(u)u$ where $c \in C((0, \infty), (0, \infty))$ and $\frac{\partial u}{\partial n}$ is the outward normal derivative of $u$ on $|x| = r_0$. Here $K \in C^1([r_0, \infty), (0, \infty))$ satisfies $\lim_{r \to \infty} K(r) = 0$, and $f \in C^1(0, \infty)$ is strictly increasing and satisfies $f(0) < 0$, $\lim_{s \to \infty} f(s) = \infty$, $\lim_{s \to \infty} \frac{f(s)}{s^{p-1}} = 0$ and $\frac{f(s)}{s^q}$ is nonincreasing on $[a, \infty)$ for some $a > 0$ and $q \in (0, p-1)$. We establish uniqueness results for positive radial solutions for $\lambda \gg 1$. (Received September 14, 2016)