In our recent paper, we studied the reduced D-Kaup–Newell spectral problems from $sl(2,\mathbb{R})$, and established a hierarchy of commuting bi-Hamiltonian soliton equations by zero curvature equations and we also explicitly computed their hereditary recursion operators.

In this paper, we discuss a new spectral problem: $\phi_x = U(\lambda,p,q)\phi$ which possesses the same soliton hierarchy as the reduced D-Kaup–Newell spectral problems. With the aid of the Lax matrix, we introduce an hyperelliptic curve of arithmetic genus $n$: $\mathcal{K}_n = \{(\lambda,y) : y^2 - R(\lambda) = 0\}$, where $R$ is a monic polynomial of degree $2n + 2$.

In order to formulate algebro-geometric solutions to the soliton hierarchies in terms of the Riemann theta functions, we develop a scheme to determine Dubrovin type equations for zeros and poles of meromorphic functions. We straighten out all flows in soliton hierarchies under the Abel-Jacobi coordinates associated with Lax pairs, and study the asymptotic behaviors of the Baker-Akhiezer functions. Finally, we succeed in finding the theta function representations of the potentials $p$ and $q$. (Received September 20, 2016)