The planar $n$-vortex problem is a Hamiltonian system of differential equations that effectively approximates vorticity evolution in fluid dynamics. Because collisions between point vortices must be excluded, the topology of the underlying configuration space is non-trivial and Morse theory can be applied to yield interesting results. We identify an explicit connection between the Morse index of a critical point of the Hamiltonian restricted to a level surface of the angular impulse and the eigenvalues of the corresponding relative equilibrium periodic solution (a rigidly rotating configuration). The Morse inequalities are then used to classify the stability (and instability) of all relative equilibria in some particular cases of the four-vortex problem. (Received September 19, 2016)