It is known from the work of Brunel and Aaronson, Lin, and Weiss that if $T$ is a conservative ergodic nonsingular transformation without a finite invariant measure, then there exists a conservative ergodic infinite measure-preserving K-automorphism $S$ such that $T \times S$ is not conservative. It is known that conservative ergodic infinite measure-preserving K-automorphism cannot be rigid. We prove that for any countable collection of nonsingular transformation $\{T_n\}$ with no equivalent finite invariant measure, there exists a rank-one (hence conservative ergodic) transformation $S$ such that $T_n \times S$ is not conservative for each $n$, and that moreover $S$ can be chosen to be rigid or have infinite ergodic index. We also study related questions for infinite measure-preserving $\mathbb{Z}^d$-actions and flows. (Received September 20, 2016)