We formulate a mathematical theory of frame multiplication, in which two essential algebraic operations can be made compatible in a natural way. The motivation comes from our approach to defining vector-valued ambiguity functions, that in turn are formulated to provide realistic modelling of multi-sensor environments in which a useful time-frequency analysis is required.

The technology underlying frame multiplication theory is the theory of frames, short time Fourier transforms (STFTs), and the representation theory of finite groups. The functions whose ambiguity function we wish to define have a given finite group $G$ as their domain, e.g., $G = \mathbb{Z}/N\mathbb{Z}$. Our results have the following form: i. if frame multiplication exists in the context of the aforementioned operations, then the vector-valued ambiguity function is well-defined; ii. frame multiplication exists if and only if the finite frames that arise in the theory are of a certain type, e.g., harmonic frames or, more generally, group frames. (Received July 08, 2016)