We study relations between probability measures $\mu$ on the unit circle and their sequences of Verblunsky coefficients $\alpha = \{\alpha_n\}_{n=0}^{\infty}$ (which are coefficients in the recurrence relation obeyed by orthogonal polynomials with respect to $\mu$).

The Szegő theorem is a celebrated result giving a necessary and sufficient integral criterion for $\mu$ to have $\alpha \in \ell^2$. Higher-order Szegő theorems are similar equivalence statements involving weaker decay, and bounded variation, conditions on $\alpha$. We will discuss results which test Simon’s conjecture for the general form of these theorems, in the multifrequency regime and in the regime of very slow decay. (Received September 04, 2016)