Hypersurfaces with central convex cross-sections.

The compact transverse cross-sections of a cylinder over a central ovaloid in \( \mathbb{R}^n \), \( n \geq 3 \), with hyperplanes are central ovaloids. A similar result holds for quadrics (level sets of quadratic polynomials in \( \mathbb{R}^n \), \( n \geq 3 \)). Their compact transverse cross-sections with hyperplanes are ellipsoids, which are central ovaloids.

In \( \mathbb{R}^3 \), Blaschke, Brunn, and Olovjanischnikoff found results for compact, convex surfaces that motivated B. Solomon to prove that these two kinds of examples provide the only complete, connected, smooth surfaces in \( \mathbb{R}^3 \), whose ovaloid cross-sections are central. We generalize that result to all higher dimensions, proving: If \( M^{n-1} \subseteq \mathbb{R}^n \), \( n \geq 4 \), is a complete, connected, smooth hypersurface, which intersects at least one hyperplane transversally along an ovaloid, and every such ovaloid on \( M \) is central, then \( M \) is either a cylinder over a central ovaloid or a quadric. (Received July 03, 2016)