We introduce the last-passage percolation model (LLP) and relate it to several much-studied stochastic models such as the corner growth model, queues in series, and the totally asymmetric simple exclusion process. By letting two infections from different seeds compete for space, the growth model turns into one of competition. We show how certain Busemann function limits can be proved with the help of results from queuing theory. These functions carry information about the large scale behavior of the system. They provide equations for the limiting shape and can be used to prove existence, uniqueness, and coalescence of geodesics, under mild regularity assumptions on the shape. Busemann functions are also useful to study the interface between the two growing infections. Planar directed LPP has the advantage over undirected first passage percolation in that it has an exactly solvable case that provides a window to the deeper properties of the entire class of models. This is the case where vertex weights are exponentially or geometrically distributed. The distribution of the Busemann functions becomes then fairly explicit. This leads to a number of precise results, such as closed-form expressions for the limit shape function and for the limiting angle of the competition interface. (Received September 21, 2016)