The order of the fluctuations of the passage time from the origin to a distant vertex $x$ in first-passage percolation (FPP) has been the subject of intense investigation. It is believed that the variance grows as $\|x\|^{2\chi}$, for $\chi > 0$. In two dimensions, the exponent $\chi$ is expected to be typical of the KPZ universality class: $\chi = 1/3$. This has been confirmed rigorously for some special models of last passage percolation, but remains mysterious in general.

In these lectures, I will discuss bounds on the order of fluctuations in FPP in $\mathbb{Z}^d$, for general $d$, focusing on two results. First, exponential concentration on a linear scale for the passage times, as proved by Kesten. Secondly, I will present an estimate of order $\|x\|_1 / \log \|x\|_1$ for the variance. Such a bound was first derived by Benaimi, Kalai and Schramm (BKS) for Bernoulli edge weights. The BKS result was generalized to other weight distributions, first by Benaim-Rossignol, and then Damron, Hanson, and myself. Although far from optimal, this sublinear bound remains the best known to date. I will explain how these results follow from supplementing concentration results with simple observations about the global effect of varying the passage time across single edges. (Received September 28, 2016)