In first-passage percolation (FPP), a finite geodesic between vertices $x$ and $y$ is an optimizing path for the random metric $T(\cdot, \cdot)$. An infinite geodesic is an infinite path whose finite subpaths are finite geodesics. There are many natural questions about the structure of infinite geodesics: the number of distinct infinite geodesics, whether they are asymptotically directed, and whether a doubly infinite geodesic can exist.

Much can be said about these questions under an unproven curvature assumption on the model’s limiting shape—e.g., that every infinite geodesic has an asymptotic direction. Busemann functions were brought to FPP as a tool for proving similar statements under minimal assumptions. For instance, they have been used to prove that there exist more than two disjoint infinite geodesics without any unverified assumptions.

Busemann functions have the form $B(x, y) = \lim_k[T(x, z_k) - T(y, z_k)]$ for $(z_k)_k$ the vertices on an infinite geodesic. $B(x, y)$ encodes the relative favorability of the points $x$ and $y$ for infecting $z_k$ for $k$ large, and so the asymptotics of $B$ control the direction of geodesics to $(z_k)$. We will discuss the limiting behavior of Busemann functions and its relationship to properties of infinite FPP geodesics. (Received September 30, 2016)