Lewis Carroll once published a set of mathematical “pillow problems”; brainteasers designed to give the reader something to ponder before going to sleep. One of the problems was this: what is the probability that a randomly chosen triangle is obtuse? This talk will explore a surprising connection between the shapes of polygons and the Grassmann manifold of 2-planes in real or complex n-space. Using this bridge, a number of interesting problems about random polygons and shapes have recently been solved, including explicit computations of some expectations for geometric properties of random polygons and fast algorithms for polygon sampling. The talk will be accessible to a wide mathematical audience, including students, and will also sketch some connections with biology and polymer science. (Received May 23, 2016)