How many $L$-functions are there?

$L$-functions and their cousins known as automorphic forms are borne out of, and play a central role in our understanding of, arithmetic problems ranging from the distribution of prime numbers and elliptic curves to partition numbers, quadratic forms, equidistribution of geodesics on arithmetic surfaces, and many more. This affectionately nicknamed “zoo” is a rich landscape that is often best studied in large naturally occurring families of related objects. The size of such a family in turn acts as its universal essential characteristic in analytic approaches to $L$-functions such as moment evaluations. In this talk, which will emphasize the underlying intuition, we will address the problem of counting automorphic forms and $L$-functions, surveying both the classical asymptotic results and our recent joint work with Farrell Brumley. (Received September 20, 2016)